Displacement and Velocity

Key Ideas

- The one-dimensional time-dependent kinematic equations of the previous chapter may be modified by replacing each position, velocity and acceleration with its vector equivalent.
- The position function $\vec{r}(t)$ gives the position as a function of time of a particle moving in two or three dimensions. Graphically, it is a vector from the origin of a chosen coordinate system to the point where the particle is located at a specific time.
- Motion in any given direction is independent of motion in a perpendicular direction.

Learning Objectives

After completing this section, you should be able to...

- apply the vector-version of the kinematic equations where appropriate or the scalar-component version of the kinematic equations where appropriate,
- calculate position vectors in a multidimensional displacement problem,
- solve for the displacement in two or three dimensions,
- calculate the velocity vector given the position vector as a function of time, and
- calculate the average velocity in multiple dimensions.

Displacement and velocity in two or three dimensions are straightforward extensions of the one-dimensional definitions. However, now they are vector quantities, so calculations with them have to follow the rules of vector algebra, not scalar algebra. This section builds very strongly on our earlier work with one-dimensional kinematics, so this development should proceed more quickly while reviewing and enhancing our previous understanding. Let's assume that we are always working in more than one dimension, so our vector notation will be very explicit, not implied.

To describe motion in two and three dimensions, we must similarly first establish a coordinate system and a convention for the axes. In the image below, a cyclist on a track moves from the earlier position $\vec{r}_1$ to the later position $\vec{r}_2$ as defined relative to the position of a woman who is observing. The change in the position of the cyclist is the displacement labeled $\overrightarrow{d}_{1\to 2}$. From the image, we can see that vector addition gives $\vec{r}_2 = \vec{r}_1 + \vec{d}_{1\to 2}$. The displacement is then $\vec{d}_{1\to 2} = \vec{r}_2 - \vec{r}_1 = \Delta\vec{r}$.

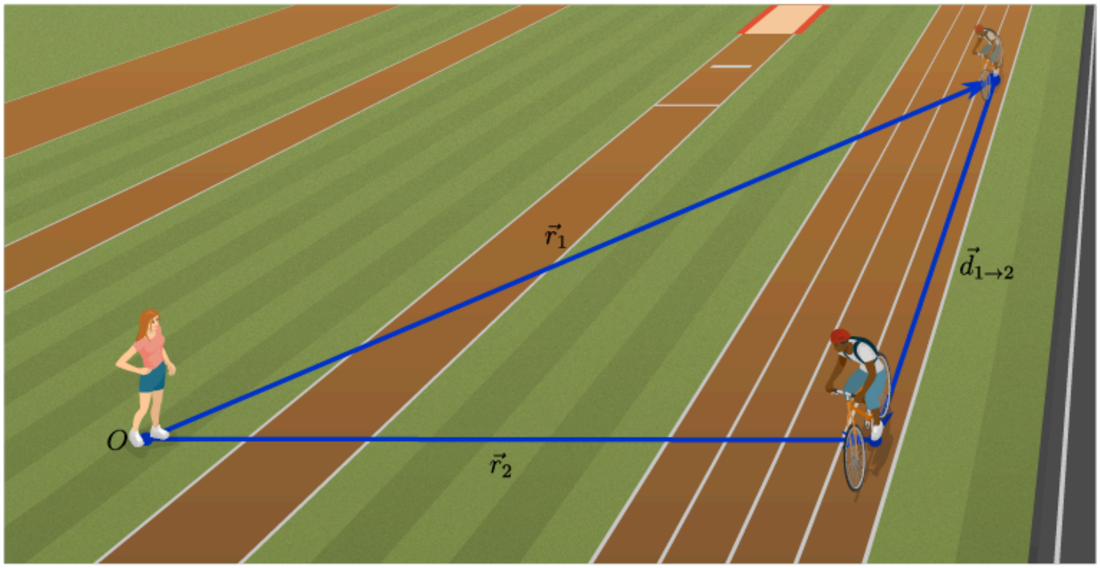

Figure 4.2 A stationary observer is standing at the origin while watching a cyclist move from position $\vec{r}_1$ to position $\vec{r}_2$ along a track that is perpendicular to the x-axis. Vector $\vec{r}_2$ is parallel to the x-axis.

Of course, it takes time for the cyclist to make that displacement. So, let's suppose that at time t the cyclist is at position $\vec{r}_1$, then after an elapsed time Δt, the cyclist is at position $\vec{r}_2$. The displacement can now be expressed as follows:

$$\vec{d} = \Delta\vec{r} = \vec{r}(t + \Delta t) - \vec{r}(t) \tag{4.1}$$

We generally use the coordinates x, y, and z to locate a particle at point $P(x, y, z)$ in three dimensions. If the particle is moving, the variables x, y, and z are functions of time:

$$x = x(t) \quad y = y(t) \quad z = z(t) \tag{4.2}$$

The **position vector** from the origin of the coordinate system to point $\mathbf{P}$ is $\vec{r}(t)$. In unit vector notation, introduced in Coordinate Systems and Vector Components, $\vec{r}(t)$ is

$$\vec{r}(t) = x(t)\hat{i} + y(t)\hat{j} + z(t)\hat{k} \tag{4.3}$$

Figure 4.2 shows the right-handed coordinate system and the vector to the point $\mathbf{P}$, where a particle could be located at a particular time t. Note the orientation of the x, y, and z axes.

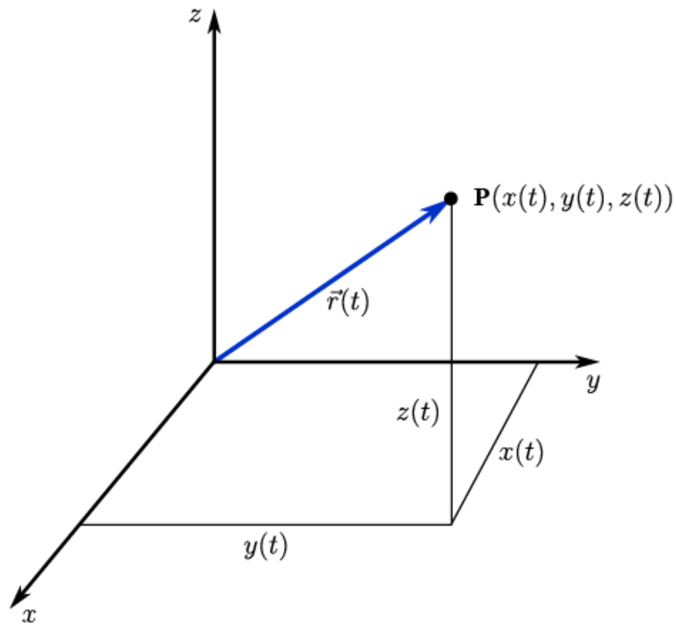

Figure 4.3 A three-dimensional coordinate system is shown with a particle at position $\mathbf{P}(x(t), y(t), z(t))$.

Now that we have a means to clearly communicate the position of an object, we can expand our language to discuss how the position of an object changes. A child at a playground, for example, might be at the entrance at one moment in time and at the base of a slide some time later. Although the child may have moved all over the playground between those two time points, her parents might want to know how far away she currently is from the entrance. The child's "displacement" from the slide to the entrance is the path showing the direct route she would travel to return to the entrance. More generally, in three dimensions, we can use our definition of the position of a particle in three-dimensional space to formulate the three-dimensional displacement. Figure 4.3 shows a particle at time t_1 located at P_1 with position vector $\vec{r}(t_1)$. At a later time t_2 the particle is located at P_2 with position vector $\vec{r}(t_2)$. The displacement vector is found by subtracting $\vec{r}(t_1)$ from $\vec{r}(t_2)$.

The following examples illustrate the concept of displacement in multiple dimensions.

Example 4.1

Weather Satellite

Consider a satellite that is collecting data for weather forecasting that is in a circular polar orbit around Earth at an altitude of $400\,\text{km}$—meaning, it passes directly overhead at the North and South Poles. What is the magnitude and direction of the displacement vector from when it is directly over the North Pole to when it is at $\theta = -45^\circ$ latitude?

Strategize

We make a picture of the problem to visualize the solution graphically. This will aid in our understanding of the displacement. We then use unit vectors to solve for the displacement. Figure 4.3 shows the surface of Earth and a circle that represents the orbit of the satellite. Although satellites are moving in three-dimensional space, they follow elliptical trajectories, which can be graphed in two dimensions. The position vectors are drawn from the center of Earth, which we take to be the origin of the coordinate system, with the y-axis as north and the x-axis as east. The vector between them is the displacement of the satellite. We take the radius of Earth as $6370\,\text{km}$, so the magnitude of each position vector is $r = 6770\,\text{km}$.

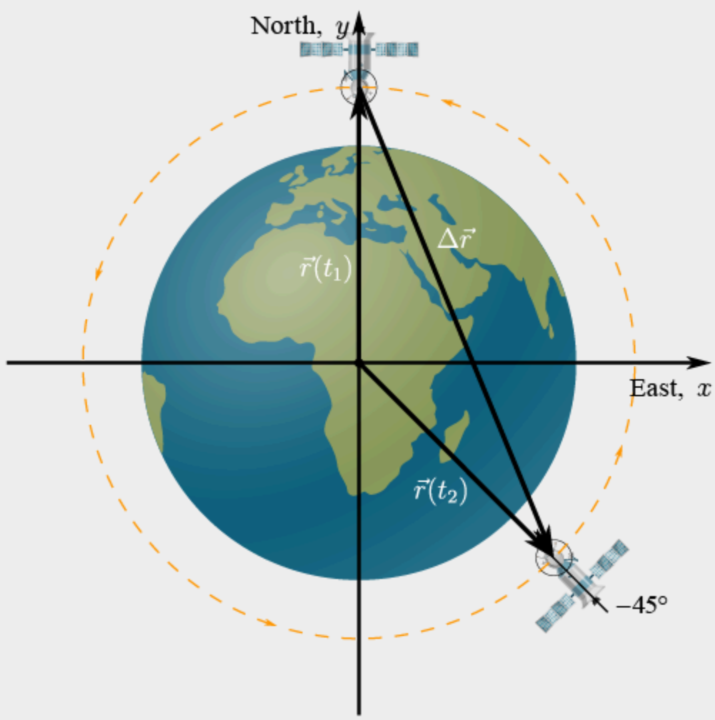

Figure 4.4 Two position vectors are drawn from the center of the Earth, which is the origin of the coordinate system, with the y-axis as north and the x-axis as east. The vector between them is the displacement of the satellite.

Develop and Solve

In unit vector notation, the position vectors are